

;;; Deliverable D17 (V2.1) in KIF

;;;28 May 03

;THIS IS A TRANSLATION IN **KIF** (ACCORDING TO THE KIF-DRAFT
;PROPOSED TO THE AMERICAN NATIONAL STANDARD NCITS.T2/98-004
;http://logic.stanford.edu/kif/dpans.html) OF THE DELIVERABLE D17 V2.1 of which
;it should be considered an appendix.
;For aknowldegments, please check the main document: Deliverable D17
;For comments on this version, please contact: stborgo@indiana.edu

REVIEW INFO

CHANGES (changes in comments or due to changes in the Deliverable D17 are not reported)	COMMENTS
(D13): changed WORD into WORLD	Typo
(NA3)-(NA9) have been dropped	These occur already somewhere else
(NA10)-(NA12) are left as comments	These are guaranteed by def. (ND5)
(NA13) has been dropped	It follows from (NA14) and (D2)

; Basic functions and relations

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; new non-rigid universals introduced in specialized theories or in new versions
; of DOLCE need to be added in this definition as new disjunction clauses of
; form (= ?f ...)
; (ND1) universals
(defrelation UNIVERSAL (?f) :=
  (or (X ?f)))

; []() iff [] [] []
; new rigid universals introduced in new versions of DOLCE (or by the user) need to
; be added in this definition
; (ND2) rigid universals
(defrelation X (?f) :=
  (or (= ?f ALL) (= ?f AB) (= ?f R) (= ?f TR) (= ?f T) (= ?f PR) (= ?f S)
    (= ?f AR) (= ?f Q) (= ?f TQ) (= ?f TL) (= ?f PQ) (= ?f SL) (= ?f AQ)
    (= ?f ED) (= ?f M) (= ?f PED) (= ?f F) (= ?f POB) (= ?f APO)
    (= ?f NAPO) (= ?f NPED) (= ?f NPOB) (= ?f MOB) (= ?f SOB) (= ?f ASO)
    (= ?f SAG) (= ?f SC) (= ?f NASO) (= ?f AS) (= ?f PD) (= ?f EV)
    (= ?f ACH) (= ?f ACC) (= ?f STV) (= ?f ST) (= ?f PRO)))

; there are no particulars in this version of DOLCE, any particular has to be
; added in this definition, the def. will have form : (or (= ?x ...) (= ?x ...))
; (ND3) particulars
(defrelation PARTICULAR(?x) :=
  )

; there are no named worlds in this version of DOLCE, any world has to be
; added in this definition, the def. will have form : (or (= ?w ...) (= ?w ...))
; (ND4) worlds
(defrelation WORLD(?w) :=
  )

; (ND5) accessibility relation on worlds
(defrelation WLDR(?w ?v) :=
  (and (WORLD ?w) (WORLD ?v)))

; (ND6) Parthood
(defrelation P (?w ?x ?y) :=>
  (and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y)))

; (ND7) Temporal Parthood
(defrelation P (?w ?x ?y ?t) :=>
  (and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y) (PARTICULAR ?t)))

; (ND8) Constitution
(defrelation K (?w ?x ?y ?t) :=>
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(and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y) (PARTICULAR ?t)))

; (ND9) Participation
(defrelation PC (?w ?x ?y ?t) :=>
  (and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y) (PARTICULAR ?t)))

; (ND10) Quality
(defrelation qt (?w ?x ?y) :=>
  (and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y)))

; (ND11) Quale
(defrelation ql (?w ?x ?y) :=>
  (and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y)))

; (ND12) Quale (temporal)
(defrelation ql (?w ?x ?y ?t) :=>
  (and (WORLD ?w) (PARTICULAR ?x) (PARTICULAR ?y) (PARTICULAR ?t)))

;*****

; (NA1) NEW AXIOM: total domain
(forall (?x)
  (or (PARTICULAR ?x) (UNIVERSAL ?x) (WORLD ?x)))

; (NA2) partition of the domain
(forall (?x)
  (and (<=> (PARTICULAR ?x)
    (and (not (UNIVERSAL ?x)) (not (WORLD ?x))))
    (<=> (UNIVERSAL ?x)
    (and (not (PARTICULAR ?x)) (not (WORLD ?x))))
    (<=> (WORLD ?x)
    (and (not (PARTICULAR ?x)) (not (UNIVERSAL ?x)))))))

; Formal Characterization
;PRINCIPLES USED IN THE TRANSLATION IN KIF:
;Modal operators of possibility and necessity are translated in the standard
; way, see ad es. p516 of Handbook of Logic in AI and Logic Prog. Vol.4;
; ex.  $\Box\Box(x)$  becomes  $\Box v(Rwv \Box (v,x))$  where v ranges over the worlds
;The indices of relations are included prefixing a dot (we preserve the capital or
; lower case distinction)
;These are the only predicates (with their arity) that do not have possible worlds
; as arguments:
; X_1,PARTICULAR_1,UNIVERSAL_1, =_2

;No need for Barcan formulas, the domain of particulars turns out to be unique
; in the translation

;WLDR is an equivalence relation (from correspondence theory, this implies
; that WLDR is a relation for S5). The axioms (NA10)-(NA12) are not necessary
; because of our definition of WLDR.
; (NA10)
;(forall (?w0) (=> (WORLD ?w0) (WLDR ?w0 ?w0)))
; (NA11)
;(forall (?w0 ?w1)
;  (=> (and (WLDR ?w0 ?w1) (WORLD ?w0) (WORLD ?w1))
;    (WLDR ?w1 ?w0)))
; (NA12)
;(forall (?w0 ?w1 ?w2)
;  (=> (and (WLDR ?w0 ?w1)
;    (WLDR ?w1 ?w2)
;    (WORLD ?w0)
;    (WORLD ?w1)
;    (WORLD ?w2))
;    (WLDR ?w0 ?w2))))

; ***THE UNIVERSALS ARE NECESSARILY NON-EMPTY***-- axiom

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; (NA14)  $\Box\Box(\text{NEP}(\Box))$ -- axiom
(forall (?w ?f) (=> (and (UNIVERSAL ?f) (WORLD ?w))
  (NEP ?w ?f)))

; (NA15)  $\Box\Box(\Box(\Box \text{RG}(\Box))$ -- axiom
(forall (?w ?f) (=> (and (UNIVERSAL ?f) (WORLD ?w))
  (or (not (X ?f)) (RG ?w ?f)))))

; (NA16) Instances of PT — axiom
(forall (?w0) (=> (WORLD ?w0)
  (and (PT ?w0 ALL ED PD Q AB)
    (PT ?w0 ED PED NPED AS)
    (PT ?w0 PED M F POB)
    (PT ?w0 POB APO NAPO)
    (PT ?w0 NPOB MOB SOB)
    (PT ?w0 SOB ASO NASO)
    (PT ?w0 ASO SAG SC)
    (PT ?w0 PD EV STV)
    (PT ?w0 EV ACH ACC)
    (PT ?w0 STV ST PRO)
    (PT ?w0 Q TQ PQ AQ)
    (PT ?w0 R TR PR AR)))))

; (NA17) Instances of SB -- axiom
(forall (?w0)
  (=> (WORLD ?w0)
    (and (SB ?w0 ALL ED) (SB ?w0 ALL PD) (SB ?w0 ALL Q) (SB ?w0 ALL AB)
      (SB ?w0 ED PED) (SB ?w0 ED NPED) (SB ?w0 ED AS)
      (SB ?w0 PED M) (SB ?w0 PED F) (SB ?w0 PED POB)
      (SB ?w0 POB APO) (SB ?w0 POB NAPO)
      (SB ?w0 NPED NPOB)
      (SB ?w0 NPOB MOB) (SB ?w0 NPOB SOB)
      (SB ?w0 SOB ASO) (SB ?w0 SOB NASO)
      (SB ?w0 ASO SAG) (SB ?w0 ASO SC)
      (SB ?w0 PD EV) (SB ?w0 PD STV)
      (SB ?w0 EV ACH) (SB ?w0 EV ACC)
      (SB ?w0 STV ST) (SB ?w0 STV PRO)
      (SB ?w0 Q TQ) (SB ?w0 Q PQ) (SB ?w0 Q AQ)
      (SB ?w0 TQ TL)
      (SB ?w0 PQ SL)
      (SB ?w0 AB FACT) (SB ?w0 AB SET) (SB ?w0 AB R)
      (SB ?w0 R TR) (SB ?w0 R PR) (SB ?w0 R AR)
      (SB ?w0 TR T)
      (SB ?w0 PR S)))))

; (NA18) Existence of sum
(forall (?w0 ?x ?y)
  (=> (and (PARTICULAR ?x) (PARTICULAR ?y) (WORLD ?w0))
    (exists (?z)
      (and (PARTICULAR ?z) (+ ?w0 ?x ?y ?z)))))

; (NA19) Existence of sigma
(forall (?w0 ?f)
  (=> (and (UNIVERSAL ?f) (WORLD ?w0))
    (exists (?z)
      (and (PARTICULAR ?z) (sigma ?w0 ?f ?z)))))

; (NA20) Existence of sum.t
(forall (?w0 ?x ?y)
  (=> (and (PARTICULAR ?x) (PARTICULAR ?y) (WORLD ?w0))
    (exists (?z)
      (and (PARTICULAR ?z) (+.t ?w0 ?x ?y ?z)))))

; (NA21) Existence of sigma.t
(forall (?w0 ?f)
  (=> (and (UNIVERSAL ?f) (WORLD ?w0))
    (exists (?z)
      (and (PARTICULAR ?z) (sigma.t ?w0 ?f ?z)))))

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; this could be added in the def. of UNIVERSAL
;(forall (@f)
;  (<=> (UNIVERSAL @f)
;       (exists (?g @h) (and (UNIVERSAL ?g)
;                             (or (UNIVERSAL @h) (= @h (listof)))
;                             (= @f (listof ?g @h))))))

; this could be added in the def. of PARTICULAR
;(forall (@x)
;  (<=> (PARTICULAR @x)
;       (exists (?y @z) (and (PARTICULAR ?y)
;                             (or (PARTICULAR @z) (= @z (listof)))
;                             (= @x (listof ?y @z))))))

;*****
;(D1) RG( $\Box$ ) =df  $\Box x(\Box(x) \Box \Box(x))$  ( $\Box$  is Rigid)
(defrelation RG (?w0 ?f) :=
  (and (UNIVERSAL ?f)
        (WORLD ?w0)
        (forall (?w ?x)
          (=> (and (WLDR ?w0 ?w) (WORLD ?w) (PARTICULAR ?x))
              (=> (?f ?w ?x)
                  (forall (?u)
                    (=> (and (WLDR ?w ?u) (WORLD ?u))
                        (?f ?u ?x))))))))

;(D2) NEP( $\Box$ ) =df  $\Box x(\Box(x))$  ( $\Box$  is Non-Empty)
(defrelation NEP (?w0 ?f) :=
  (and (UNIVERSAL ?f)
        (WORLD ?w0)
        (forall (?w)
          (=> (and (WLDR ?w0 ?w) (WORLD ?w))
              (exists (?y)
                (and (PARTICULAR ?y) (?f ?w ?y)))))))

;(D3) DJ( $\Box, \Box$ ) =df  $\Box x(\Box(x) \Box \Box(x))$  ( $\Box$  and  $\Box$  are Disjoint)
(defrelation DJ (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (forall (?w ?x)
          (=> (and (WLDR ?w0 ?w)
                  (WORLD ?w)
                  (PARTICULAR ?x))
              (not (and (?f ?w ?x) (?g ?w ?x)))))))

;(D4) SB( $\Box, \Box$ ) =df  $\Box x(\Box(x) \Box \Box(x))$  ( $\Box$  Subsumes  $\Box$ )
(defrelation SB (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (forall (?w ?x)
          (=> (and (WLDR ?w0 ?w)
                  (WORLD ?w)
                  (PARTICULAR ?x))
              (or (not (?g ?w ?x)) (?f ?w ?x))))))

;(D5) EQ( $\Box, \Box$ ) =df SB( $\Box, \Box$ )  $\Box$  SB( $\Box, \Box$ ) ( $\Box$  and  $\Box$  are Equal)
(defrelation EQ (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f) (UNIVERSAL ?g) (WORLD ?w0) (SB ?w0 ?f ?g) (SB ?w0 ?g ?f)))

;(D6) PSB( $\Box, \Box$ ) =df SB( $\Box, \Box$ )  $\Box \Box$  SB( $\Box, \Box$ ) ( $\Box$  Properly Subsumes  $\Box$ )
(defrelation PSB (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f) (UNIVERSAL ?g) (WORLD ?w0) (SB ?w0 ?f ?g)
        (not (SB ?w0 ?f ?g))))
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; (D7)  $L(\Box) =_{df} \Box \Box (SB(\Box, \Box) \Box EQ(\Box, \Box))$  ( $\Box$  is a Leaf)
(defrelation L (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (forall (?w ?g)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (UNIVERSAL ?g))
        (or (not (?SB ?w0 ?f ?g)) (EQ ?w0 ?f ?g)))))))

; (D8)  $SBL(\Box, \Box) =_{df} SB(\Box, \Box) \Box L(\Box)$  ( $\Box$  is a Leaf Subsumed by  $\Box$ )
(defrelation SBL (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f) (UNIVERSAL ?g) (WORLD ?w0) (SB ?w0 ?f ?g) (L ?w0 ?g)))

; (D9)  $PSBL(\Box, \Box) =_{df} PSB(\Box, \Box) \Box L(\Box)$  ( $\Box$  is a Leaf Properly Subsumed by  $\Box$ )
(defrelation PSBL (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f) (UNIVERSAL ?g) (WORLD ?w0) (PSB ?w0 ?f ?g) (L ?w0 ?g)))

; (D10)  $L_{\Box}(\Box) =_{df} \Box(\Box) \Box \Box \Box ((SB(\Box, \Box) \Box \Box(\Box) \Box EQ(\Box, \Box))$  ( $\Box$  is a Leaf in  $\Box$ )
(defrelation L.X (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (X ?f)
    (forall (?w ?g)
      (=> (and (WLDR ?w0 ?w) (WORLD ?w) (UNIVERSAL ?g))
        (=> (and (?SB ?w ?f ?g) (X ?g))
          (EQ ?w ?f ?g)))))))

; (D11)  $SBL_{\Box}(\Box, \Box) =_{df} SB(\Box, \Box) \Box L_{\Box}(\Box)$ 
(defrelation SBL.X (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f) (UNIVERSAL ?g) (WORLD ?w0) (SB ?w0 ?f ?g) (L.X ?w0 ?g)))

; (D12)  $PSBL_{\Box}(\Box, \Box) =_{df} PSB(\Box, \Box) \Box L_{\Box}(\Box)$ 
(defrelation PSBL.X (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f) (UNIVERSAL ?g) (WORLD ?w0) (PSB ?w0 ?f ?g) (L.X ?w0 ?g)))

; Definition (D13) is left for expressivity. In practice it becomes superfluous
; since the user needs to give a list of the n-tuple satisfying relation PT in
; axiom (NA17)
; (D13)  $PT(\Box, \Box_1, \dots, \Box_n) =_{df} \Box \neq \Box_i \Box DJ(\Box, \Box_j)$  for  $1 \leq i \neq j \leq n$ 
;  $\Box \Box x(\Box(x) \Box (\Box_1(x) \dots \Box_n(x)))$  ( $\Box_1, \dots, \Box_n$  is a Partition of  $\Box$ )
(defrelation PT (?w0 ?f @g) :=
  (and (UNIVERSAL ?f)
    (UNIVERSAL @g)
    (WORLD ?w0)
    (not (item ?f @g))
    (forall (?h ?k)
      (and (=> (and (UNIVERSAL ?h)
        (UNIVERSAL ?k)
        (item ?h @g)
        (item ?k @g)
        (/= ?h ?k))
        (DJ ?w0 ?h ?k))
      (forall (?w ?x)
        (=> (and (WLDR ?w0 ?w)
          (WORLD ?w)
          (PARTICULAR ?x))
          (<=> (?f ?w ?x)
            (exists (?h)
              (and (UNIVERSAL ?h)
                (item ?h @g)
                (?h ?w ?x))))))))))

; Mereological Definitions
; (D14)  $PP(x, y) =_{df} P(x, y) \Box \Box P(y, x)$  (Proper Part)
(defrelation PP (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)

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(PARTICULAR ?y)
(WORLD ?w0)
(P ?w0 ?x ?y)
(not (P ?w0 ?y ?x)))

; (D15)  $O(x, y) =_{df} \exists z (P(z, x) \wedge P(z, y))$  (Overlap)
(defrelation O (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (exists (?z) (and (PARTICULAR ?z)
                          (P ?w0 ?z ?x)
                          (P ?w0 ?z ?y)))))

; (D16)  $At(x) =_{df} \neg \exists y (PP(y, x))$  (Atom)
(defrelation At (?w0 ?x) :=
  (and (PARTICULAR ?x)
        (WORLD ?w0)
        (not (exists (?y) (and (PARTICULAR ?y)
                              (PP ?w0 ?y ?x))))))

; (D17)  $AtP(x, y) =_{df} P(x, y) \wedge At(x)$  (Atomic Part)
(defrelation AtP (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (P ?w0 ?x ?y)
        (At ?w0 ?x)))

; (D18)  $x + y =_{df} \exists w (O(w, z) \wedge (O(w, x) \vee O(w, y)))$  (Binary Sum)
(defrelation + (?w0 ?x ?y ?z) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?z)
        (WORLD ?w0)
        (forall (?u)
          (=> (PARTICULAR ?u)
              (<=> (O ?w0 ?u ?z)
                  (or (O ?w0 ?u ?x) (O ?w0 ?u ?y))))))
        (forall (?z1)
          (=> (and (PARTICULAR ?z1)
                  (forall (?u)
                    (=> (PARTICULAR ?u)
                        (<=> (O ?w0 ?u ?z1)
                            (or (O ?w0 ?u ?x) (O ?w0 ?u ?y))))))
              (= ?z1 ?z)))))

; (D19)  $\Box x \Box (x) =_{df} \Box y (O(y, z) \wedge \Box w (\Box (w) \wedge O(y, w)))$  (Sum of  $\Box$ 's)
; Note: the rendition in KIF is weaker than the corresponding definition in
; modal FOL; here ?f has to be one of the universal introduced explicitly.
; [A possible way out: use string-variables (@f) to code Boolean
; combinations of universals.]
(defrelation sigma (?w0 ?f ?z) :=
  (and (PARTICULAR ?z)
        (UNIVERSAL ?f)
        (WORLD ?w0)
        (forall (?y)
          (=> (PARTICULAR ?y)
              (<=> (O ?w0 ?y ?z)
                  (exists (?v)
                    (and (PARTICULAR ?v)
                        (?f ?w0 ?v)
                        (O ?w0 ?y ?v))))))
        (forall (?z1)
          (=> (PARTICULAR ?z1)
              (exists (?y)
                (and (PARTICULAR ?y)
                    (=> (<=> (O ?w0 ?y ?z1)
                        (or (O ?w0 ?y ?z)
                            (O ?w0 ?y ?z1)))))))))

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        (exists (?v)
          (and (PARTICULAR ?v)
              (?f ?w0 ?v)
              (O ?w0 ?y ?v))))))
    (= ?z1 ?z))))))

; (D20) PP(x, y, t) =df P(x, y, t) ∧ ∧ P(y, x, t)  (Temporary Proper Part)
(defrelation PP (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (P ?w0 ?x ?y ?t)
        (not (P ?w0 ?y ?x ?t))))

; (D21) O(x, y, t) =df ∃z (P(z, x, t) ∧ P(z, y, t))  (Temporary Overlap)
(defrelation O (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?z) (and (PARTICULAR ?z)
                          (P ?w0 ?z ?x ?t)
                          (P ?w0 ?z ?y ?t)))))

; (D22) At(x, t) =df ∃y (PP(y, x, t))  (Temporary Atom)
(defrelation At (?w0 ?x ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (not (exists (?y)
                     (and (PARTICULAR ?y) (PP ?w0 ?y ?x ?t))))))

; (D23) AtP(x, y, t) =df P(x, y, t) ∧ At(x, t)  (Temporary Atomic Part)
(defrelation AtP (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (P ?w0 ?x ?y ?t)
        (At ?w0 ?x ?t)))

; (D24) x =t y =df P(x, y, t) ∧ P(y, x, t)  (Coincidence)
(defrelation =.t (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (P ?w0 ?x ?y ?t)
        (P ?w0 ?y ?x ?t)))

; (D25) CP(x, y) =df ∃t (PRE(y, t) ∧ ∃t (PRE(y, t) ∧ P(x, y, t))  (Constant Part)
(defrelation CP (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (exists (?t)
          (and (PARTICULAR ?t) (PRE ?w0 ?y ?t)))
        (forall (?t)
          (=> (and (PARTICULAR ?t) (PRE ?w0 ?y ?t))
              (P ?w0 ?x ?y ?t)))))

; (D26) x +t y =df ∃w, t (O(w, z, t) ∧ (O(w, x, t) ∧ O(w, y, t)))
(defrelation +.t (?w0 ?x ?y ?z) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?z)

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(WORLD ?w0)
(forall (?u ?t)
  (=> (and (PARTICULAR ?u) (PARTICULAR ?t))
    (<=> (O ?w0 ?u ?z ?t)
      (or (O ?w0 ?u ?x ?t) (O ?w0 ?u ?y ?t))))))
(forall (?z1 ?t)
  (=> (and (PARTICULAR ?z1)
    (PARTICULAR ?t)
    (forall (?u)
      (=> (PARTICULAR ?u)
        (<=> (O ?w0 ?u ?z1 ?t)
          (or (O ?w0 ?u ?x ?t) (O ?w0 ?u ?y ?t))))))
    (= ?z1 ?z))))))

; (D27)  $\Box x \Box (x) =_{df} \Box y, t (O(y, z, t) \Box \Box w (\Box (w) \Box O(y, w, t)))$ 
; NOTE: this rendition includes only the listed universal, for instance,
; no Boolean combination of universals is included [see also comment on (D19)]
(defrelation sigma.t (?w0 ?f ?z) :=
  (and (PARTICULAR ?z)
    (UNIVERSAL ?f)
    (WORLD ?w0)
    (forall (?y ?t)
      (=> (and (PARTICULAR ?y) (PARTICULAR ?t))
        (<=> (O ?w0 ?y ?z ?t)
          (exists (?v)
            (and (PARTICULAR ?v)
              (?f ?w0 ?v)
              (O ?w0 ?y ?v ?t))))))
    (forall (?z1 ?t)
      (=> (and (PARTICULAR ?z1) (PARTICULAR ?t))
        (exists (?y)
          (and (PARTICULAR ?y)
            (=> (<=> (O ?w0 ?y ?z1 ?t)
              (exists (?v)
                (and (PARTICULAR ?v)
                  (?f ?w0 ?v)
                  (O ?w0 ?y ?v ?t))))
            (= ?z1 ?z))))))))))

; Quality
; (D28)  $dqt(x, y) =_{df} qt(x, y) \Box \Box z (qt(x, z) \Box qt(z, y))$  (Direct Quality)
(defrelation dqt (?w0 ?x ?y) :=
  (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (qt ?w0 ?x ?y)
    (not (exists (?z)
      (and (PARTICULAR ?z)
        (qt ?w0 ?x ?z)
        (qt ?w0 ?z ?y))))))

; (D29)  $qt(\Box, x, y) =_{df} qt(x, y) \Box \Box (x) \Box SBL_{\Box}(Q, \Box)(Quality\ of\ type\ \Box)$ 
(defrelation qtf (?w0 ?f ?x ?y) :=
  (and (UNIVERSAL ?f)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (qt ?w0 ?x ?y)
    (?f ?w0 ?x)
    (SBL.X ?w0 Q ?f)))

; Temporal and Spatial Quale
; (D30)  $ql_{T,PD}(t, x) =_{df} PD(x) \Box \Box z (qt(TL, z, x) \Box ql(t, z))$ 
(defrelation ql.T.PD (?w0 ?t ?x) :=
  (and (PARTICULAR ?t)
    (PARTICULAR ?x)
    (WORLD ?w0)
    (PD ?w0 ?x)

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(exists (?z) (and (PARTICULAR ?z)
                  (qtf ?w0 TL ?z ?x)
                  (ql ?w0 ?t ?z))))))

; (D31)  $ql_{T,ED}(t, x) =_{df} ED(x) \wedge t = \lfloor t' \rfloor \vee (PC(x, y, t'))$ 
(defrelation ql.T.ED (?w0 ?t ?x) :=
  (and (PARTICULAR ?t)
        (PARTICULAR ?x)
        (WORLD ?w0)
        (ED ?w0 ?x)
        (forall (?u)
          (=> (PARTICULAR ?u)
              (<=> (O ?w0 ?u ?t)
                  (exists (?v ?y)
                    (and (PARTICULAR ?v)
                        (PARTICULAR ?y)
                        (PC ?w0 ?x ?y ?v)
                        (O ?w0 ?u ?v)))))))
        (forall (?t1)
          (=> (PARTICULAR ?t1)
              (exists (?u)
                (and (PARTICULAR ?u)
                    (=> (<=> (O ?w0 ?u ?t1)
                        (exists (?v ?y)
                          (and (PARTICULAR ?v)
                              (PARTICULAR ?y)
                              (PC ?w0 ?x ?y ?v)
                              (O ?w0 ?u ?v))))))
                  (= ?t1 ?t))))))))))

; (D32)  $ql_{T,TQ}(t, x) =_{df} TQ(x) \wedge \exists z (qt(x, z) \wedge ql_{T,PD}(t, z))$ 
(defrelation ql.T.TQ (?w0 ?t ?x) :=
  (and (PARTICULAR ?t)
        (PARTICULAR ?x)
        (WORLD ?w0)
        (TQ ?w0 ?x)
        (exists (?z) (and (PARTICULAR ?z)
                          (qtf ?w0 ?x ?z)
                          (ql.T.PD ?w0 ?t ?z))))))

; (D33)  $ql_{T,PQ} AQ(t, x) =_{df} (PQ(x) \wedge AQ(x)) \wedge \exists z (qt(x, z) \wedge ql_{T,ED}(t, z))$ 
(defrelation ql.T.PQAQ (?w0 ?t ?x) :=
  (and (PARTICULAR ?t)
        (PARTICULAR ?x)
        (WORLD ?w0)
        (or (PQ ?w0 ?x) (AQ ?w0 ?x))
        (exists (?z) (and (PARTICULAR ?z)
                          (qtf ?w0 ?x ?z)
                          (ql.T.ED ?w0 ?t ?z))))))

; (D34)  $ql_{T,Q}(t, x) =_{df} ql_{T,TQ}(t, x) \vee ql_{T,PQ} AQ(t, x)$ 
(defrelation ql.T.Q (?w0 ?t ?x) :=
  (and (PARTICULAR ?t)
        (PARTICULAR ?x)
        (WORLD ?w0)
        (or (ql.T.TQ ?w0 ?t ?x)
            (ql.T.PQAQ ?w0 ?t ?x))))

; (D35)  $ql_T(t, x) =_{df} ql_{T,ED}(t, x) \vee ql_{T,PD}(t, x) \vee ql_{T,Q}(t, x)$  (Temporal Quale)
(defrelation ql.T (?w0 ?t ?x) :=
  (and (PARTICULAR ?t)
        (PARTICULAR ?x)
        (WORLD ?w0)
        (or (ql.T.ED ?w0 ?t ?x)
            (ql.T.PD ?w0 ?t ?x)
            (ql.T.Q ?w0 ?t ?x))))

; (D36)  $ql_{S,PED}(s, x, t) =_{df} PED(x) \wedge \exists z (qt(sL, z, x) \wedge ql(s, z, t))$ 

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(defrelation ql.S.PED (?w0 ?s ?x ?t) :=
  (and (PARTICULAR ?s)
        (PARTICULAR ?x)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (PED ?w0 ?x)
        (exists (?z) (and (PARTICULAR ?z)
                           (qtf ?w0 SL ?z ?x)
                           (ql ?w0 ?s ?z ?t))))))

; (D37)  $ql_{S,PQ}(s, x, t) =_{df} PQ(x) \wedge \exists z (qt(x, z) \wedge ql_{S,PED}(s, z, t))$ 
(defrelation ql.S.PQ (?s ?x ?t) :=
  (and (PARTICULAR ?s)
        (PARTICULAR ?x)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (PQ ?w0 ?x)
        (exists (?z) (and (PARTICULAR ?z)
                           (qt ?w0 ?x ?z)
                           (ql.S.PED ?w0 ?s ?z ?t))))))

; (D38)  $ql_{S,PD}(s, x, t) =_{df} PD(x) \wedge \exists z (mppc(z, x) \wedge ql_{S,PED}(s, z, t))$ 
(defrelation ql.S.PD (?w0 ?s ?x ?t) :=
  (and (PARTICULAR ?s)
        (PARTICULAR ?x)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (PD ?w0 ?x)
        (exists (?z) (and (PARTICULAR ?z)
                           (mppc ?w0 ?z ?x)
                           (ql.S.PED ?w0 ?s ?z ?t))))))

; (D39)  $ql_S(s, x, t) =_{df} ql_{S,PED}(s, x, t) \vee ql_{S,PQ}(s, x, t) \vee ql_{S,PD}(s, x, t)$  (Spatial Quale)
(defrelation ql.S (?w0 ?s ?x ?t) :=
  (and (PARTICULAR ?s)
        (PARTICULAR ?x)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (or (ql.S.PED ?w0 ?s ?x ?t)
            (ql.S.PQ ?w0 ?s ?x ?t)
            (ql.S.PD ?w0 ?s ?x ?t))))

; Being present
; (D40)  $PRE(x, t) =_{df} \exists t' (ql_T(t', x) \wedge P(t, t'))$  (Being Present at t)
(defrelation PRE (?w0 ?x ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?u) (and (PARTICULAR ?u)
                           (ql.T ?w0 ?u ?x)
                           (P ?w0 ?t ?u))))))

; (D41)  $PRE(x, s, t) =_{df} PRE(x, t) \wedge \exists s' (ql_S(s', x, t) \wedge P(s, s'))$  (Being Present in s at t)
(defrelation PRE (?w0 ?x ?s ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?s)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (PRE ?w0 ?x ?t)
        (exists (?u) (and (PARTICULAR ?u)
                           (ql.S ?w0 ?u ?x ?t)
                           (P ?w0 ?s ?u))))))

; Inclusion and Coincidence
; (D42)  $x \sqsubseteq_T y =_{df} \exists t, t' (ql_T(t, x) \wedge ql_T(t', y) \wedge P(t, t'))$  (Temporal Inclusion)
(defrelation incl.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)

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(WORLD ?w0)
(exists (?t ?u) (and (PARTICULAR ?t)
                     (PARTICULAR ?u)
                     (ql.T ?w0 ?t ?x)
                     (ql.T ?w0 ?u ?y)
                     (P ?w0 ?t ?u))))))

; (D43)  $x \sqsubseteq_T y =_{df} \exists t, t' (ql_T(t, x) \sqcap ql_T(t', y) \sqcap PP(t, t'))$  (Proper Temporal Inclusion)
(defrelation incl.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (exists (?t ?u) (and (PARTICULAR ?t)
                              (PARTICULAR ?u)
                              (ql.T ?w0 ?t ?x)
                              (ql.T ?w0 ?u ?y)
                              (PP ?w0 ?t ?u))))))

; (D44)  $x \sqsubseteq_{S,t} y =_{df} \exists s, s' (ql_S(s, x, t) \sqcap ql_S(s', y, t) \sqcap P(s, s'))$  (Temporary Spatial Inclusion)
(defrelation incl.S.t (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?s ?r) (and (PARTICULAR ?s)
                              (PARTICULAR ?r)
                              (ql.S ?w0 ?s ?x ?t)
                              (ql.S ?w0 ?r ?y ?t)
                              (P ?w0 ?s ?r))))))

; (D45)  $x \sqsubseteq_{S,t} y =_{df} \exists s, s' (ql_S(s, x, t) \sqcap ql_S(s', y, t) \sqcap PP(s, s'))$  (Temp. Proper Sp. Inclusion)
(defrelation incl.S.t (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?s ?r) (and (PARTICULAR ?s)
                              (PARTICULAR ?r)
                              (ql.S ?w0 ?s ?x ?t)
                              (ql.S ?w0 ?r ?y ?t)
                              (PP ?w0 ?s ?r))))))

; (D46)  $x \sqsubseteq_{ST} y =_{df} \exists t (PRE(x, t) \sqcap \exists t' (PRE(x, t) \sqcap x \sqsubseteq_{S,t'} y))$  (Spatio-temporal Inclusion)
(defrelation incl.S.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (exists (?t) (and (PARTICULAR ?t) (PRE ?w0 ?x ?t)))
        (forall (?t) (=> (and (PARTICULAR ?t) (PRE ?w0 ?x ?t))
                        (incl.S.t ?w0 ?x ?y ?t)))))

; (D47)  $x \sqsubseteq_{ST,t} y =_{df} PRE(x, t) \sqcap \exists t' (AtP(t', t) \sqcap x \sqsubseteq_{S,t'} y)$  (Spatio-temp. Incl. during t)
(defrelation incl.S.T.t (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (PRE ?w0 ?x ?t)
        (forall (?u) (=> (and (PARTICULAR ?u) (AtP ?w0 ?u ?t))
                        (incl.S.t ?w0 ?x ?y ?u)))))

; (D48)  $x \sqsubseteq_T y =_{df} (x \sqsubseteq_T y \sqcap y \sqsubseteq_T x)$  (Temporal Coincidence)
(defrelation ~.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (incl.T ?w0 ?x ?y)
        (incl.T ?w0 ?y ?x)))

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; (D49)  $x \sqsubseteq_{S,t} y =_{df} (x \sqsubseteq_{S,t} y \sqcap y \sqsubseteq_{S,t} x)$  (Temporary Spatial Coincidence)
(defrelation ~.S.t (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (incl.S.t ?w0 ?x ?y ?t)
        (incl.S.t ?w0 ?y ?x ?t)))

; (D50)  $x \sqsubseteq_{ST} y =_{df} (x \sqsubseteq_{ST} y \sqcap y \sqsubseteq_{ST} x)$  (Spatio-temporal Coincidence)
(defrelation ~.S.T (?w0 ?x ?y) :=
  (and (WORLD ?w0)
        (PARTICULAR ?x)
        (PARTICULAR ?y)
        (incl.S.T ?w0 ?x ?y)
        (incl.S.T ?w0 ?y ?x)))

; (D51)  $x \sqsubseteq_{ST,t} y =_{df} PRE(x, t) \sqcap t'(AtP(t', t) \sqcap x \sqsubseteq_{S,t'} y)$  (Spatio-temp. Coincidence dur. t)
(defrelation ~.S.T.t (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (PRE ?w0 ?x ?t)
        (forall (?u) (=> (and (PARTICULAR ?u) (AtP ?w0 ?u ?t))
                          (~.S.t ?w0 ?x ?y ?u)))))

; (D52)  $x \circ_T y =_{df} \sqcap t'(ql_T(t, x) \sqcap ql_T(t', y) \sqcap O(t, t'))$  (Temporal Overlap)
(defrelation O.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (exists (?t ?u) (and (PARTICULAR ?t)
                             (PARTICULAR ?u)
                             (ql.T ?w0 ?t ?x)
                             (ql.T ?w0 ?u ?y)
                             (O ?w0 ?t ?u)))))

; (D53)  $x \circ_{S,t} y =_{df} \sqcap s,s'(ql_S(s, x, t) \sqcap ql_S(s', y, t) \sqcap O(s, s'))$  (Temporary Spatial Overlap)
(defrelation O.S.t (?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?s ?r) (and (PARTICULAR ?s)
                             (PARTICULAR ?r)
                             (ql.S ?w0 ?s ?x ?t)
                             (ql.S ?w0 ?r ?y ?t)
                             (O ?w0 ?s ?r)))))

; Perdurant

; (D54)  $P_T(x, y) =_{df} PD(x) \sqcap P(x, y) \sqcap \sqcap z((P(z, y) \sqcap z \sqsubseteq_T x) \sqcap P(z, x))$  (Temporal Part)
(defrelation P.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (WORLD ?w0)
        (PD ?w0 ?x)
        (P ?w0 ?x ?y)
        (forall (?z) (=> (and (PARTICULAR ?z)
                             (P ?w0 ?z ?y)
                             (incl.T ?w0 ?z ?x))
                          (P ?w0 ?z ?x)))))

; (D55)  $P_S(x, y) =_{df} PD(x) \sqcap P(x, y) \sqcap x \sqsubseteq_T y$  (Spatial Part)
(defrelation P.S (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)

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(PARTICULAR ?y)
(WORLD ?w0)
(PD ?w0 ?x)
(P ?w0 ?x ?y)
(~.T ?w0 ?x ?y))

; (D56) NEPs(□) =df SB(PD, □) □ □□x,y((□(x) □ □(y) □ □P(x,y) □ □P(y,x))
; (□ is Strongly Non-Empty)
(defrelation NEP.S (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w) (=> (and (WLDR ?w0 ?w) (WORLD ?w))
      (exists (?x ?y)
        (and (PARTICULAR ?x)
          (PARTICULAR ?y)
          (?f ?w ?x)
          (?f ?w ?y)
          (not (P ?w ?x ?y))
          (not (P ?w ?y ?x))))))))))

; (D57) CM(□) =df SB(PD, □) □ □□x,y((□(x) □ □(y)) □ □(x+y)) (□ is Cumulative)
(defrelation CM (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w ?x ?y ?z)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?z)
        (+ ?w ?x ?y ?z)
        (?f ?w ?x)
        (?f ?w ?y))
        (?f ?w ?z))))))

; (D58) CM~(□) =df SB(PD, □) □ □□x,y((□(x) □ □(y) □ □P(x,y) □ □P(y,x)) □ □□(x+y))
; (□ is Anti-Cumulative)
(defrelation CM~ (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w ?x ?y ?z)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?z)
        (+ ?w ?x ?y ?z)
        (?f ?w ?x)
        (?f ?w ?y)
        (not (P ?w ?x ?y))
        (not (P ?w ?y ?x)))
        (not (?f ?w ?z))))))

; (D59) HOM(□) =df SB(PD, □) □ □□x,y((□(x) □ PT(y, x)) □ □(y)) (□ is Homeomeric)
(defrelation HOM (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w ?x ?y) (=> (and (WLDR ?w0 ?w)
      (WORLD ?w)
      (PARTICULAR ?x)
      (PARTICULAR ?y)
      (?f ?w ?x)
      (P.T ?w ?y ?x))
      (?f ?w ?y))))))

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;(D60) HOM~( $\square$ ) =df SB(PD,  $\square$ )  $\square$   $\square$   $\square$  x( $\square$ (x)  $\square$   $\square$  v(PT(y, x)  $\square$   $\square$   $\square$ (y)) ( $\square$  is Anti-Homeom.)
(defrelation HOM~ (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w ?x)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
        (?f ?w ?x))
      (exists (?y)
        (and (PARTICULAR ?y)
          (P.T ?w ?y ?x)
          (not (?f ?w ?y))))))))

;(D61) AT( $\square$ ) =df SB(PD,  $\square$ )  $\square$   $\square$   $\square$  x( $\square$ (x)  $\square$  At(x)) ( $\square$  is Atomic)
(defrelation AT (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w ?x) (=> (and (WLDR ?w0 ?w)
      (WORLD ?w)
      (PARTICULAR ?x)
      (?f ?w ?x))
      (At ?w ?x)))))

;(D62) AT~( $\square$ ) =df SB(PD,  $\square$ )  $\square$   $\square$   $\square$  x( $\square$ (x)  $\square$   $\square$  At(x)) ( $\square$  is Anti-Atomic)
(defrelation AT~ (?w0 ?f) :=
  (and (UNIVERSAL ?f)
    (WORLD ?w0)
    (SB ?w0 PD ?f)
    (forall (?w ?x) (=> (and (WLDR ?w0 ?w)
      (WORLD ?w)
      (PARTICULAR ?x)
      (?f ?w ?x))
      (not (At ?w ?x))))))

;Participation
;(D63) PCC(x, y) =df  $\square$  t(PRE(y, t))  $\square$   $\square$  t(PRE(y, t)  $\square$  PC(x, y, t)) (Constant Participation)
(defrelation PC.C (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (exists (?t) (and (PARTICULAR ?t) (PRE ?w0 ?y ?t)))
    (forall (?t) (=> (and (PARTICULAR ?t)
      (PRE ?w0 ?y ?t))
      (PC ?w0 ?x ?y ?t)))))

;(D64) PCT(x, y, t) =df PD(y)  $\square$   $\square$  z((P(z, y)  $\square$  PRE(z, t))  $\square$  PC(x, z, t)) (Temporary Total Particip.)
(defrelation PC.T (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (WORLD ?w0)
    (PD ?w0 ?y)
    (forall (?z)
      (=> (and (PARTICULAR ?z)
        (P ?w0 ?z ?y)
        (PRE ?w0 ?z ?t))
        (PC ?w0 ?x ?z ?t)))))

;(D65) PCT(x, y) =df  $\square$  t(q!t(t, y)  $\square$  PCT(x, y, t)) (Total Participation)
(defrelation PC.T (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (exists (?t) (and (PARTICULAR ?t)

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(q1.T ?w0 ?t ?y)
(PC.T ?w0 ?x ?y ?t))))))

; (D66) mpc(x, y) =df x =  $\exists z$ (PCT(z, y)) (Maximal Participant)
(defrelation mpc (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (forall (?z ?t)
      (=> (and (PARTICULAR ?z) (PARTICULAR ?t))
        (<=> (O ?w0 ?z ?x ?t)
          (exists (?v)
            (and (PARTICULAR ?v)
              (PC.T ?w0 ?v ?y ?t)
              (O ?w0 ?z ?v ?t)))))))
    (forall (?z ?x1 ?t)
      (=> (and (PARTICULAR ?z)
        (PARTICULAR ?x1)
        (PARTICULAR ?t)
        (<=> (O ?w0 ?z ?x1 ?t)
          (exists (?v)
            (and (PARTICULAR ?v)
              (PC.T ?w0 ?v ?y ?t)
              (O ?w0 ?z ?v ?t)))))))
      (= ?x1 ?x))))))

; (D67) mppc(x, y) =df x =  $\exists z$ (PCT(z, y)  $\wedge$  PED(z)) (Maximal Physical Participant)
(defrelation mppc (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (forall (?z ?t)
      (=> (and (PARTICULAR ?z) (PARTICULAR ?t))
        (<=> (O ?w0 ?z ?x ?t)
          (exists (?v)
            (and (PARTICULAR ?v)
              (PC.T ?w0 ?v ?y ?t)
              (PED ?w0 ?z)
              (O ?w0 ?z ?v ?t)))))))
    (forall (?z ?x1 ?t)
      (=> (and (PARTICULAR ?z)
        (PARTICULAR ?x1)
        (PARTICULAR ?t)
        (<=> (O ?w0 ?z ?x1 ?t)
          (exists (?v)
            (and (PARTICULAR ?v)
              (PC.T ?w0 ?v ?y ?t)
              (PED ?w0 ?z)
              (O ?w0 ?z ?v ?t)))))))
      (= ?x1 ?x))))))

; (D68) lf(x, y) =df x =  $\exists z$ (PCT(y, z)) (Life)
(defrelation lf (?w0 ?x ?y) :=
  (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (forall (?z)
      (=> (PARTICULAR ?z)
        (<=> (O ?w0 ?z ?x)
          (exists (?v)
            (and (PARTICULAR ?v)
              (PC.T ?w0 ?y ?v)
              (O ?w0 ?z ?v)))))))
    (forall (?z ?u)
      (=> (and (PARTICULAR ?z) (PARTICULAR ?u)
        (<=> (O ?w0 ?z ?u)
          (exists (?v)
            (and (PARTICULAR ?v)
              (PC.T ?w0 ?y ?v)
              (O ?w0 ?z ?v)))))))
      (= ?x ?u))))))

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(PC.T ?w0 ?y ?v)
(O ?w0 ?z ?v))))))
(= ?u ?x))))))

; Dependence
;(D69) SD(x,y)=df  $\Box(\Box t(\text{PRE}(x,t)) \Box \Box t(\text{PRE}(x,t) \Box \text{PRE}(y,t)))$  (Specific Constant Dep.)
(defrelation SD (?w0 ?x ?y) :=
  (or (and (PARTICULAR ?x)
    (PARTICULAR ?y)
    (WORLD ?w0)
    (forall (?w)
      (=> (and (WLDR ?w0 ?w) (WORLD ?w))
        (and (exists (?t)
          (and (PARTICULAR ?t) (PRE ?w ?x ?t)))
          (forall (?t)
            (=> (and (PARTICULAR ?t) (PRE ?w ?x ?t))
              (PRE ?w ?y ?t))))))))))
    (and (UNIVERSAL ?x)
      (UNIVERSAL ?y)
      (WORLD ?w0)
      (DJ ?w0 ?x ?y)
      (forall (?w ?x1)
        (=> (and (WLDR ?w0 ?w)
          (WORLD ?w)
          (PARTICULAR ?x1)
          (?x ?w ?x1))
          (exists (?y1) (and (PARTICULAR ?y1)
            (?y ?w ?y1)
            (SD ?w ?x1 ?y1))))))))))

;(D70) SD( $\Box$ ,  $\Box$ )=df DJ( $\Box$ ,  $\Box$ )  $\Box \Box \Box x(\Box(x) \Box \Box y(\Box(y) \Box \text{SD}(x,y)))$  (Specific Const. Dep.)
;included in def (D69)

;(D71) GD( $\Box$ ,  $\Box$ )=df DJ( $\Box$ ,  $\Box$ )  $\Box \Box (\Box x(\Box(x) \Box \Box t(\text{PRE}(x,t)) \Box \Box x,t(\Box(x) \Box \text{At}(t) \Box \text{PRE}(x,t)) \Box \Box y(\Box(y) \Box \text{PRE}(y,t))))$  (Generic Const. Dep.)
(defrelation GD (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (WORLD ?w0)
    (DJ ?w0 ?f ?g)
    (forall (?w ?x ?t)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
        (PARTICULAR ?t)
        (?f ?w ?x))
        (and (exists (?t1)
          (and (PARTICULAR ?t1) (PRE ?w ?x ?t1)))
          (=> (and (At ?w ?t) (PRE ?w ?x ?t))
            (exists (?y)
              (and (PARTICULAR ?y)
                (?g ?w ?y)
                (PRE ?w ?y ?t))))))))))

;(D72) D( $\Box$ ,  $\Box$ )=df SD( $\Box$ ,  $\Box$ ) GD( $\Box$ ,  $\Box$ )(Constant Dependence)
(defrelation D (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (WORLD ?w0)
    (or (SD ?w0 ?f ?g) (GD ?w0 ?f ?g))))

;(D73) OD( $\Box$ ,  $\Box$ )=df D( $\Box$ ,  $\Box$ )  $\Box \Box \Box D(\Box, \Box)$  (One-sided Constant Dependence)
(defrelation OD (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (WORLD ?w0)
    (D ?w0 ?f ?g)
    (not (D ?w0 ?g ?f))))

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; (D74) OSD( $\square, \square$ ) =df SD( $\square, \square$ )  $\square \square$  D( $\square, \square$ ) (One-sided Specific Constant Dependence)
(defrelation OSD (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (SD ?w0 ?f ?g)
        (not (D ?w0 ?g ?f))))

; (D75) OGD( $\square, \square$ ) =df GD( $\square, \square$ )  $\square \square$  D( $\square, \square$ ) (One-sided Generic Constant Dependence)
(defrelation OGD (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GD ?w0 ?f ?g)
        (not (D ?w0 ?g ?f))))

; (D76) MSD( $\square, \square$ ) =df SD( $\square, \square$ )  $\square$  SD( $\square, \square$ ) (Mutual Specific Constant Dependence)
(defrelation MSD (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (SD ?w0 ?f ?g)
        (SD ?w0 ?g ?f)))

; (D77) MGD( $\square, \square$ ) =df GD( $\square, \square$ )  $\square$  GD( $\square, \square$ ) (Mutual Generic Constant Dependence)
(defrelation MGD (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GD ?w0 ?f ?g)
        (GD ?w0 ?g ?f)))

; Spatial Dependence
; (D78) SDS( $x, y$ ) =df  $\square(\square t, s(\text{PRE}(x, s, t)) \square \square s, t(\text{PRE}(x, s, t) \square \text{PRE}(y, s, t)))$ 
; (Specific Spatial Dependence)
(defrelation SD.S (?w0 ?x ?y) :=
  (or (and (WORLD ?w0)
            (PARTICULAR ?x)
            (PARTICULAR ?y)
            (forall (?w)
              (=> (and (WLDLDR ?w0 ?w) (WORLD ?w)
                      (and (exists (?t ?s)
                          (and (PARTICULAR ?t)
                              (PARTICULAR ?s)
                              (PRE ?w ?x ?s ?t))))
                    (forall (?t ?s)
                      (=> (and (PARTICULAR ?t)
                              (PARTICULAR ?s)
                              (PRE ?w ?x ?s ?t))
                        (PRE ?w ?y ?s ?t)))))))
      (and (WORLD ?w0)
            (UNIVERSAL ?x)
            (UNIVERSAL ?y)
            (DJ ?w0 ?x ?y)
            (forall (?w ?x1)
              (=> (and (WLDLDR ?w0 ?w)
                      (WORLD ?w)
                      (PARTICULAR ?x1)
                      (?x ?w ?x))
                  (exists (?y1)
                    (and (PARTICULAR ?y1)
                        (?y ?w ?y1)
                        (SD.S ?w ?x1 ?y1))))))))))

; (D79) PSDS( $x, y$ ) =df  $\square(\square t, s(\text{PRE}(x, s, t)) \square \square s, t(\text{PRE}(x, s, t) \square \square s'(\text{PP}(s', s) \square \text{PRE}(y, s', t))))$ 
; (Partial Specific Spatial Dependence)
(defrelation PSD.S (?w0 ?x ?y) :=

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(or (and (WORLD ?w0)
  (PARTICULAR ?x)
  (PARTICULAR ?y)
  (forall (?w)
    (=> (and (WLDR ?w0 ?w) (WORLD ?w))
      (and (exists (?t ?s)
        (and (PARTICULAR ?t)
          (PARTICULAR ?s)
          (PRE ?w ?x ?s ?t)))
        (forall (?t ?s)
          (=> (and (PARTICULAR ?t)
            (PARTICULAR ?s)
            (PRE ?w ?x ?s ?t)))
          (exists (?r)
            (and (PARTICULAR ?r)
              (PP ?w ?r ?s)
              (PRE ?w ?y ?r ?t))))))))))
  (and (WORLD ?w0)
    (UNIVERSAL ?x)
    (UNIVERSAL ?y)
    (DJ ?w0 ?x ?y)
    (forall (?w ?x1)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x1)
        (?x ?w ?x1))
        (exists (?y1)
          (and (PARTICULAR ?y1)
            (?y ?w ?y1)
            (PSD.S ?w ?x1 ?y1))))))))))

; (D80)  $P^{-1}SD_S(x, y) =_{df} \Box(\Box_{t,s}(PRE(x, s, t)) \Box \Box_{s,t}(PRE(x, s, t) \Box \Box_{s'}(PP(s, s') \Box PRE(y, s', t))))$ 
; (Inverse Partial Specific Spatial Dependence)
(defrelation P1SD.S (?w0 ?x ?y) :=
  (or (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (forall (?w)
      (=> (and (WLDR ?w0 ?w) (WORLD ?w))
        (and (exists (?t ?s)
          (and (PARTICULAR ?t)
            (PARTICULAR ?s)
            (PRE ?w ?x ?s ?t)))
          (forall (?t ?s)
            (=> (and (PARTICULAR ?t)
              (PARTICULAR ?s)
              (PRE ?w ?x ?s ?t)))
            (exists (?r)
              (and (PARTICULAR ?r)
                (PP ?w ?s ?r)
                (PRE ?w ?y ?r ?t))))))))))
    (and (WORLD ?w0)
      (UNIVERSAL ?x)
      (UNIVERSAL ?y)
      (DJ ?w0 ?x ?y)
      (forall (?w ?x1)
        (=> (and (WLDR ?w0 ?w)
          (WORLD ?w)
          (PARTICULAR ?x1)
          (?x ?w ?x1))
          (exists (?y1)
            (and (PARTICULAR ?y1)
              (?y ?w ?y1)
              (P1SD.S ?w ?x1 ?y1))))))))))

; (D81)  $SD_S(\Box, \Box) =_{df} DJ(\Box, \Box) \Box \Box \Box x(\Box(x) \Box \Box y(\Box(y) \Box SD_S(x, y)))$ 
; included in def (D78)

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; (D82) PSDS( $\Box$ ,  $\Box$ ) =df DJ( $\Box$ ,  $\Box$ )  $\Box$   $\Box$   $\Box$  x( $\Box$ (x)  $\Box$   $\Box$  y( $\Box$ (y)  $\Box$  PSDS(x, y)))
; included in def (D79)

; (D83) P-1SDS( $\Box$ ,  $\Box$ ) =df DJ( $\Box$ ,  $\Box$ )  $\Box$   $\Box$   $\Box$  x( $\Box$ (x)  $\Box$   $\Box$  y( $\Box$ (y)  $\Box$  P-1SDS(x, y)))
; included in def (D80)

; (D84) GDS( $\Box$ ,  $\Box$ ) =df DJ( $\Box$ ,  $\Box$ )  $\Box$   $\Box$  ( $\Box$  x( $\Box$ (x)  $\Box$   $\Box$  t,s(PRE(x, s, t))  $\Box$ 
;  $\Box$  x,s,t( $\Box$ (x)  $\Box$  At(t)  $\Box$  PRE(x, s, t))  $\Box$   $\Box$  y( $\Box$ (y)  $\Box$  PRE(y, s, t))) (Generic Spatial Dependence)
(defrelation GD.S (?w0 ?f ?g) :=
  (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (DJ ?w0 ?f ?g)
    (forall (?w ?x ?s ?t)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
        (PARTICULAR ?t)
        (PARTICULAR ?s)
        (?f ?w ?x))
        (and (exists (?t1 ?s1)
          (and (PARTICULAR ?t1)
            (PARTICULAR ?s1)
            (PRE ?w ?x ?s1 ?t1)))
          (=> (and (At ?w ?t) (PRE ?w ?x ?s ?t))
            (exists (?y)
              (and (PARTICULAR ?y)
                (?g ?w ?y)
                (PRE ?w ?y ?s ?t))))))))))

; (D85) PGDS( $\Box$ ,  $\Box$ ) =df DJ( $\Box$ ,  $\Box$ )  $\Box$   $\Box$  ( $\Box$  x( $\Box$ (x)  $\Box$   $\Box$  t,s(PRE(x, s, t))  $\Box$   $\Box$  x,s,t( $\Box$ (x)  $\Box$  At(t)  $\Box$ 
; PRE(x, s, t))  $\Box$   $\Box$  y,s'( $\Box$ (y)  $\Box$  PP(s', s)  $\Box$  PRE(y, s', t))) (Partial Generic Spatial Dependence)
(defrelation PGD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (WORLD ?w0)
    (DJ ?w0 ?f ?g)
    (forall (?w ?x ?s ?t)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
        (PARTICULAR ?s)
        (PARTICULAR ?t)
        (?f ?w ?x))
        (and (exists (?s1 ?t1)
          (and (PRE ?w ?x ?s1 ?t1)
            (PARTICULAR ?s1)
            (PARTICULAR ?t1))
          (=> (and (At ?w ?t) (PRE ?w ?x ?s ?t))
            (exists (?y ?u)
              (and (PARTICULAR ?y)
                (PARTICULAR ?u)
                (?g ?w ?y)
                (PP ?w ?u ?s)
                (PRE ?w ?y ?u ?t))))))))))

; (D86) P-1GDS( $\Box$ ,  $\Box$ ) =df DJ( $\Box$ ,  $\Box$ )  $\Box$   $\Box$  ( $\Box$  x( $\Box$ (x)  $\Box$   $\Box$  t,s(PRE(x, s, t))  $\Box$   $\Box$  x,s,t( $\Box$ (x)  $\Box$  At(t)  $\Box$ 
; PRE(x, s, t))  $\Box$   $\Box$  y,s'( $\Box$ (y)  $\Box$  PP(s, s')  $\Box$  PRE(y, s', t))) (Inverse Partial Generic Spatial Dependence)
(defrelation P1GD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (WORLD ?w0)
    (DJ ?w0 ?f ?g)
    (forall (?w ?x ?s ?t)
      (=> (and (WLDR ?w0 ?w)
        (WORLD ?w)
        (PARTICULAR ?x)
```

```

(PARTICULAR ?s)
(PARTICULAR ?t)
(?f ?w ?x))
(and (exists (?t1 ?s1)
      (and (PARTICULAR ?t1)
            (PARTICULAR ?s1)
            (PRE ?w ?x ?s1 ?t1))
      (=> (and (At ?w ?t) (PRE ?w ?x ?t))
          (exists (?y ?u)
                (and (PARTICULAR ?y)
                      (PARTICULAR ?u)
                      (?g ?w ?y)
                      (PP ?w ?s ?u)
                      (PRE ?w ?y ?u ?t)))))))

; (D87) DGDS( $\Box$ ,  $\Box$ ) =df GDS( $\Box$ ,  $\Box$ )  $\Box$   $\Box$  GDS( $\Box$ ,  $\Box$ )  $\Box$  GDS( $\Box$ ,  $\Box$ )
; (Direct Generic Spatial Dependence)
(defrelation DGD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GD.S ?w0 ?f ?g)
        (not (exists (?h) (and (UNIVERSAL ?h)
                                (GD.S ?w0 ?f ?h)
                                (GD.S ?w0 ?h ?g))))))

; (D88) SDtS( $x, y, t$ ) =df SDS( $x, y$ )  $\Box$  PRE( $x, t$ ) (Temporary Specific Spatial Dependence)
(defrelation SDt.S (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (SD.S ?w0 ?x ?y)
        (PRE ?w0 ?x ?t)))

; (D89) GDtS( $x, y, t$ ) =df  $\Box$   $\Box$  ( $\Box$ ( $x$ )  $\Box$   $\Box$ ( $y$ )  $\Box$  GDS( $\Box$ ,  $\Box$ )  $\Box$   $x$   $\Box$  $s, t$   $y$ ) (Temp. Gen. Sp. Dep.)
(defrelation GDt.S (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?f ?g) (and (UNIVERSAL ?f)
                              (UNIVERSAL ?g)
                              (?f ?w0 ?x)
                              (?g ?w0 ?y)
                              (GD.S ?w0 ?f ?g)
                              (~.S.t ?w0 ?x ?y ?t)))))

; (D90) DGDtS( $x, y, t$ ) =df  $\Box$   $\Box$  ( $\Box$ ( $x$ )  $\Box$   $\Box$ ( $y$ )  $\Box$  DGDS( $\Box$ ,  $\Box$ )  $\Box$   $x$   $\Box$  $s, t$   $y$ ) (Temp. Direct Sp. Dep.)
(defrelation DGDt.S (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (exists (?f ?g) (and (UNIVERSAL ?f)
                              (UNIVERSAL ?g)
                              (?f ?w0 ?x)
                              (?g ?w0 ?y)
                              (DGD.S ?w0 ?f ?g)
                              (~.S.t ?w0 ?x ?y ?t)))))

; (D91) OSDS( $\Box$ ,  $\Box$ ) =df SDS( $\Box$ ,  $\Box$ )  $\Box$   $\Box$  D( $\Box$ ,  $\Box$ ) (One-sided Specific Spatial Dependence)
(defrelation OSD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (SD.S ?w0 ?f ?g)
        (not (D ?w0 ?g ?f))))

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; (D92) OGDS(□, □) =df GDS(□, □) □ □ D(□, □) (One-sided Generic Spatial Dependence)
(defrelation OGD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GD.S ?w0 ?f ?g)
        (not (D ?w0 ?g ?f))))

; (D93) MSDS(□, □) =df SDS(□, □) □ SDS(□, □) (Mutual Specific Spatial Dependence)
(defrelation MSD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (SD.S ?w0 ?f ?g)
        (SD.S ?w0 ?g ?f)))

; (D94) MGDS(□, □) =df GDS(□, □) □ GDS(□, □) (Mutual Generic Spatial Dependence)
(defrelation MGD.S (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GD.S ?w0 ?f ?g)
        (GD.S ?w0 ?g ?f)))

; Constitution
; (D95) DK(x, y, t) =df K(x, y, t) □ □ □ z(K(x, z, t) □ K(z, y, t)) (Direct Constitution)
(defrelation DK (?w0 ?x ?y ?t) :=
  (and (PARTICULAR ?x)
        (PARTICULAR ?y)
        (PARTICULAR ?t)
        (WORLD ?w0)
        (K ?w0 ?x ?y ?t)
        (not (exists (?z) (and (PARTICULAR ?z)
                                (K ?w0 ?x ?z ?t)
                                (K ?w0 ?z ?y ?t))))))

; (D96) SK(x, y) =df □(□t(PRE(x, t) □ □t(PRE(x, t) □ K(y, x, t)))
; (x is Constantly Specifically Constituted by y)
(defrelation SK (?w0 ?x ?y) :=
  (or (and (WORLD ?w0)
            (PARTICULAR ?x)
            (PARTICULAR ?y)
            (forall (?w)
              (=> (and (WLDR ?w0 ?w) (WORLD ?w))
                  (and (exists (?t)
                        (and (PARTICULAR ?t) (PRE ?w ?x ?t))
                        (forall (?t)
                          (=> (and (PARTICULAR ?t)
                                    (PRE ?w ?x ?t))
                              (K ?w ?y ?x ?t))))))))))
      (and (UNIVERSAL ?x)
            (UNIVERSAL ?y)
            (WORLD ?w0)
            (DJ ?w0 ?f ?g)
            (forall (?w ?x1)
              (=> (and (WLDR ?w0 ?w)
                      (WORLD ?w)
                      (PARTICULAR ?x1)
                      (?f ?w ?x1))
                  (exists (?y1)
                    (and (PARTICULAR ?y1)
                        (?y ?w ?y1)
                        (SK ?w ?x1 ?y1))))))))))

; (D97) SK(□, □) =df DJ(□, □) □ □ □ x(□(x) □ □y(□(y) □ SK(x, y)))
; (□ is Constantly Specifically Constituted by □)

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;included in def (D96)

;(D98) GK( $\Box$ ,  $\Box$ ) =df DJ( $\Box$ ,  $\Box$ )  $\Box$   $\Box$ ( $\Box$ x( $\Box$ (x)  $\Box$   $\Box$ t(PRE(x, t))  $\Box$   $\Box$ x,t( $\Box$ (x)  $\Box$  At(t)  $\Box$  PRE(x, t))  $\Box$   $\Box$ y( $\Box$ (y)  $\Box$ 
; K(y, x, t))) ( $\Box$  is Constantly Generically Constituted by  $\Box$ )
(defrelation GK (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (DJ ?w0 ?f ?g)
        (forall (?w ?x ?t)
          (=> (and (WLDR ?w0 ?w)
                  (WORLD ?w)
                  (PARTICULAR ?x)
                  (PARTICULAR ?t)
                  (?f ?w ?x))
              (and (exists (?t1)
                      (and (PARTICULAR ?t1) (PRE ?w ?x ?t1)))
                    (=> (and (At ?w ?t) (PRE ?w ?x ?t))
                        (exists (?y)
                          (and (PARTICULAR ?y)
                              (?g ?w ?y)
                              (K ?w ?y ?x ?t))))))))))

;(D99) K( $\Box$ ,  $\Box$ ) =df SK( $\Box$ ,  $\Box$ ) GK( $\Box$ ,  $\Box$ ) ( $\Box$  is Constituted by  $\Box$ )
(defrelation K (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (or (SK ?w0 ?f ?g) (GK ?w0 ?f ?g))))

;(D100) OSK( $\Box$ ,  $\Box$ ) =df SK( $\Box$ ,  $\Box$ )  $\Box$   $\Box$ K( $\Box$ ,  $\Box$ ) ( $\Box$  is One-sided Cons. Specif. Const. by  $\Box$ )
(defrelation OSK (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (SK ?w0 ?f ?g)
        (not (K ?w0 ?g ?f))))

;(D101) OGK( $\Box$ ,  $\Box$ ) =df GK( $\Box$ ,  $\Box$ )  $\Box$   $\Box$ K( $\Box$ ,  $\Box$ ) ( $\Box$  is One-sided Cons. Generic. Const. by  $\Box$ )
(defrelation OGK (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GK ?w0 ?f ?g)
        (not (K ?w0 ?g ?f))))

;(D102) MSK( $\Box$ ,  $\Box$ ) =df SK( $\Box$ ,  $\Box$ )  $\Box$  SK( $\Box$ ,  $\Box$ ) (Mutual Specific Constitution)
(defrelation MSK (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (SK ?w0 ?f ?g)
        (SK ?w0 ?g ?f)))

;(D103) MGK( $\Box$ ,  $\Box$ ) =df GK( $\Box$ ,  $\Box$ )  $\Box$  GK( $\Box$ ,  $\Box$ ) (Mutual Generic Constitution)
(defrelation MGK (?w0 ?f ?g) :=
  (and (UNIVERSAL ?f)
        (UNIVERSAL ?g)
        (WORLD ?w0)
        (GK ?w0 ?f ?g)
        (GK ?w0 ?g ?f)))

; Characterization of functions and relations
; Parthood
; Argument Restrictions
;(A1) P(x, y)  $\Box$  (AB(x) PD(x))  $\Box$  (AB(y) PD(y))
(forall (?w0 ?x ?y)
```

```

(=> (and (P ?w0 ?x ?y)
         (WORLD ?w0)
         (PARTICULAR ?x)
         (PARTICULAR ?y))
    (and (or (AB ?w0 ?x) (PD ?w0 ?x))
         (or (AB ?w0 ?y) (PD ?w0 ?y)))))

; (A2)  $P(x, y) \sqsubseteq (PD(x) \sqcap PD(y))$ 
(forall (?w0 ?x ?y)
  (=> (and (P ?w0 ?x ?y)
           (WORLD ?w0)
           (PARTICULAR ?x)
           (PARTICULAR ?y))
      (<=> (PD ?w0 ?x) (PD ?w0 ?y))))

; (A3)  $P(x, y) \sqsubseteq (AB(x) \sqcap AB(y))$ 
(forall (?w0 ?x ?y)
  (=> (and (P ?w0 ?x ?y)
           (WORLD ?w0)
           (PARTICULAR ?x)
           (PARTICULAR ?y))
      (<=> (AB ?w0 ?x) (AB ?w0 ?y))))

; (A4)  $(P(x, y) \sqcap SB(R, \square) \sqcap \square(\square)) \sqsubseteq (\square(x) \sqcap \square(y))$ 
(forall (?w0 ?x ?y ?f)
  (=> (and (WORLD ?w0)
           (PARTICULAR ?x)
           (PARTICULAR ?y)
           (UNIVERSAL ?f)
           (P ?w0 ?x ?y)
           (SB ?w0 R ?f)
           (X ?f))
      (<=> (?f ?w0 ?x) (?f ?w0 ?y))))

; Ground Axioms
; (A5)  $(AB(x) \sqcap PD(x)) \sqsubseteq P(x, x)$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0)
           (PARTICULAR ?x)
           (or (AB ?w0 ?x) (PD ?w0 ?x)))
      (P ?w0 ?x ?x)))

; (A6)  $(P(x, y) \sqcap P(y, x)) \sqsubseteq x = y$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
           (PARTICULAR ?x)
           (PARTICULAR ?y)
           (P ?w0 ?x ?y)
           (P ?w0 ?y ?x))
      (= ?x ?y)))

; (A7)  $(P(x, y) \sqcap P(y, z)) \sqsubseteq P(x, z)$ 
(forall (?w0 ?x ?y ?z)
  (=> (and (WORLD ?w0)
           (PARTICULAR ?x)
           (PARTICULAR ?y)
           (PARTICULAR ?z)
           (P ?w0 ?x ?y)
           (P ?w0 ?y ?z))
      (P ?w0 ?x ?z)))

; (A8)  $((AB(x) \sqcap PD(x)) \sqcap \square P(x, y)) \sqsubseteq \square \neg (P(z, x) \sqcap \square O(z, y))$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
           (PARTICULAR ?x)
           (PARTICULAR ?y)

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        (or (AB ?w0 ?x) (PD ?w0 ?x))
        (not (P ?w0 ?x ?y)))
    (exists (?z)
      (and (PARTICULAR ?x)
        (P ?w0 ?z ?x)
        (not (O ?w0 ?z ?y)))))

; (A9) ( $\Box x \Box (x \Box (\Box x \Box (AB(x)) \Box x \Box (PD(x)))) \Box \Box y (y = \Box x \Box (x))$ )
; Note: this version in KIF consider only the universal explicitly listed
; [see comment on (D19)]
(forall (?w0 ?f)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (exists (?x)
      (and (PARTICULAR ?x) (?f ?w0 ?x)))
    (or (forall (?x)
      (=> (and (PARTICULAR ?x) (?f ?w0 ?x))
        (AB ?w0 ?x)))
      (forall (?x)
        (=> (and (PARTICULAR ?x) (?f ?w0 ?x))
          (PD ?w0 ?x))))))
    (exists (?y)
      (and (PARTICULAR ?y) (sigma ?w0 ?f ?y)))))

; Temporary Parthood
; Argument restrictions
; (A10)  $P(x, y, t) \Box (ED(x) \Box ED(y) \Box T(t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (P ?w0 ?x ?y ?t))
    (and (ED ?w0 ?x) (ED ?w0 ?y) (T ?w0 ?t))))

; (A11)  $P(x, y, t) \Box (PED(x) \Box PED(y))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (P ?w0 ?x ?y ?t))
    (<=> (PED ?w0 ?x) (PED ?w0 ?y))))

; (A12)  $P(x, y, t) \Box (NPED(x) \Box NPED(y))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (P ?w0 ?x ?y ?t))
    (<=> (NPED ?w0 ?x) (NPED ?w0 ?y))))

; Ground Axioms
; (A13)  $(P(x, y, t) \Box P(y, z, t)) \Box P(x, z, t)$ 
(forall (?w0 ?x ?y ?z ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?z)
    (PARTICULAR ?t)
    (P ?w0 ?x ?y ?t)
    (P ?w0 ?y ?z ?t))
    (P ?w0 ?x ?z ?t)))

; (A14)  $(ED(x) \Box ED(y) \Box PRE(x, t) \Box PRE(y, t)) \Box \Box P(x, y, t) \Box \Box z (P(z, x, t) \Box \Box O(z, y, t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)

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(PARTICULAR ?x)
(PARTICULAR ?y)
(PARTICULAR ?t)
(ED ?w0 ?x)
(ED ?w0 ?y)
(PRE ?w0 ?x ?t)
(PRE ?w0 ?y ?t)
(not (P ?w0 ?x ?y ?t)))
(exists (?z)
  (and (PARTICULAR ?z)
    (P ?w0 ?z ?x ?t)
    (not (O ?w0 ?z ?y ?t)))))

; (A15) ( $\Box x \Box (x \Box \Box x (\Box (x \Box ED(x))) \Box \Box y (y = \Box x \Box (x)))$ )
; [see comment on (D19)]
(forall (?w0 ?f)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (exists (?x)
      (and (PARTICULAR ?x) (?f ?w0 ?x)))
    (forall (?x)
      (=> (and (PARTICULAR ?x) (?f ?w0 ?x))
        (ED ?w0 ?x)))))
    (exists (?y)
      (and (PARTICULAR ?y) (sigma.t ?w0 ?f ?y)))))

; Links With Other Primitives
; (A16) ( $ED(x) \Box PRE(x, t) \Box P(x, x, t)$ )
(forall (?w0 ?x ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?t)
    (ED ?w0 ?x)
    (PRE ?w0 ?x ?t))
    (P ?w0 ?x ?x ?t)))

; (A17)  $P(x, y, t) \Box (PRE(x, t) \Box PRE(y, t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (P ?w0 ?x ?y ?t))
    (and (PRE ?w0 ?x ?t) (PRE ?w0 ?y ?t)))))

; (A18)  $P(x, y, t) \Box \Box t' (P(t', t) \Box P(x, y, t'))$ 
(forall (?w0 ?x ?y ?t ?u)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (PARTICULAR ?u)
    (P ?w0 ?x ?y ?t)
    (P ?w0 ?u ?t))
    (P ?w0 ?x ?y ?u)))

; (A19) ( $PED(x) \Box P(x, y, t) \Box x \Box_{S, t} y$ )
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (PED ?w0 ?x)
    (P ?w0 ?x ?y ?t))
    (incl.S.t ?w0 ?x ?y ?t)))

; Constitution
; Argument restrictions

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; (A20)  $K(x, y, t) \sqsubseteq ((ED(x) \sqsubseteq PD(x)) \sqsubseteq (ED(y) \sqsubseteq PD(y)) \sqsubseteq T(t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t))
    (and (or (ED ?w0 ?x) (PD ?w0 ?x))
      (or (ED ?w0 ?y) (PD ?w0 ?y))
      (T ?w0 ?t))))))

; (A21)  $K(x, y, t) \sqsubseteq (PED(x) \sqsubseteq PED(y))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t))
    (<=> (PED ?w0 ?x) (PED ?w0 ?y))))))

; (A22)  $K(x, y, t) \sqsubseteq (NPED(x) \sqsubseteq NPED(y))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t))
    (<=> (NPED ?w0 ?x) (NPED ?w0 ?y))))))

; (A23)  $K(x, y, t) \sqsubseteq (PD(x) \sqsubseteq PD(y))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t))
    (<=> (PD ?w0 ?x) (PD ?w0 ?y))))))

; Ground Axioms
; (A24)  $K(x, y, t) \sqsubseteq \sqsubseteq K(y, x, t)$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t))
    (not (K ?w0 ?y ?x ?t))))))

; (A25)  $(K(x, y, t) \sqsubseteq K(y, z, t)) \sqsubseteq K(x, z, t)$ 
(forall (?w0 ?x ?y ?z ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?z)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t)
    (K ?w0 ?y ?z ?t))
    (K ?w0 ?x ?z ?t))))))

; Links with other Primitives
; (A26)  $K(x, y, t) \sqsubseteq (PRE(x, t) \sqsubseteq PRE(y, t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t))
    (PRE ?w0 ?x ?t)
    (PRE ?w0 ?y ?t))))))

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    (and (PRE ?w0 ?x ?t) (PRE ?w0 ?y ?t))))

; (A27)  $K(x, y, t) \sqsubseteq \Box t'(P(t', t) \sqsubseteq K(x, y, t'))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (<=> (K ?w0 ?x ?y ?t)
      (forall (?u)
        (=> (and (PARTICULAR ?u) (P ?w0 ?u ?t))
          (K ?w0 ?x ?y ?u)))))))

; (A28)  $(K(x, y, t) \sqsubseteq PED(x)) \sqsubseteq x \sqsubseteq_{S, t} y$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (PED ?w0 ?x)
    (K ?w0 ?x ?y ?t))
    (~.S.t ?w0 ?x ?y ?t)))

; (A29)  $(K(x, y, t) \sqsubseteq P(y', y, t)) \sqsubseteq \Box x'(P(x', x, t) \sqsubseteq K(x', y', t))$ 
(forall (?w0 ?x ?y ?y1 ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?y1)
    (PARTICULAR ?t)
    (K ?w0 ?x ?y ?t)
    (P ?w0 ?y1 ?y ?t))
    (exists (?x1)
      (and (PARTICULAR ?x1)
        (P ?w0 ?x1 ?x ?t)
        (K ?w0 ?x1 ?y1 ?t)))))

; Links between Categories
; (A30)  $GK(NAPO, M)$ 
(forall (?w0) (=> (WORLD ?w0) (GK ?w0 NAPO M)))

; (A31)  $GK(APO, NAPO)$ 
(forall (?w0) (=> (WORLD ?w0) (GK ?w0 APO NAPO)))

; (A32)  $GK(SC, SAG)$ 
(forall (?w0) (=> (WORLD ?w0) (GK ?w0 SC SAG)))

; Participation
; Argument restrictions
; (A33)  $PC(x, y, t) \sqsubseteq (ED(x) \sqsubseteq PD(y) \sqsubseteq T(t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (PC ?w0 ?x ?y ?t))
    (and (ED ?w0 ?x) (PD ?w0 ?y) (T ?w0 ?t)))))

; Existential Axioms
; (A34)  $(PD(x) \sqsubseteq PRE(x, t)) \sqsubseteq \Box y(PC(y, x, t))$ 
(forall (?w0 ?x ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?t)
    (PD ?w0 ?x)
    (PRE ?w0 ?x ?t))
    (exists (?y)
      (and (PARTICULAR ?y) (PC ?w0 ?y ?x ?t)))))

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; (A35)  $ED(x) \sqsubseteq \exists y, t (PC(x, y, t))$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (ED ?w0 ?x)
    (exists (?y ?t)
      (and (PARTICULAR ?y) (PARTICULAR ?t) (PC ?w0 ?x ?y ?t))))))

; Links with other Primitives
; (A36)  $PC(x, y, t) \sqsubseteq (PRE(x, t) \sqsubseteq PRE(y, t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (PC ?w0 ?x ?y ?t))
    (and (PRE ?w0 ?x ?t) (PRE ?w0 ?y ?t)))))

; (A37)  $PC(x, y, t) \sqsubseteq \exists t' (P(t', t) \sqsubseteq PC(x, y, t'))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t))
    (<=> (PC ?w0 ?x ?y ?t)
      (forall (?u)
        (=> (and (PARTICULAR ?u) (P ?w0 ?u ?t))
          (PC ?w0 ?x ?y ?u)))))))

; Quality
; Argument restrictions:
; (A38)  $qt(x, y) \sqsubseteq (Q(x) \sqsubseteq (Q(y) \sqsubseteq ED(y) \sqsubseteq PD(y)))$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (qt ?w0 ?x ?y))
    (and (Q ?w0 ?x)
      (or (Q ?w0 ?y) (ED ?w0 ?y) (PD ?w0 ?y))))))

; (A39)  $qt(x, y) \sqsubseteq (TQ(x) \sqsubseteq (TQ(y) \sqsubseteq PD(y)))$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (qt ?w0 ?x ?y))
    (<=> (TQ ?w0 ?x)
      (or (TQ ?w0 ?y) (PD ?w0 ?y))))))

; (A40)  $qt(x, y) \sqsubseteq (PQ(x) \sqsubseteq (PQ(y) \sqsubseteq PED(y)))$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (qt ?w0 ?x ?y))
    (<=> (PQ ?w0 ?x)
      (or (PQ ?w0 ?y) (PED ?w0 ?y))))))

; (A41)  $qt(x, y) \sqsubseteq (AQ(x) \sqsubseteq (AQ(y) \sqsubseteq NPED(y)))$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (qt ?w0 ?x ?y))
    (<=> (AQ ?w0 ?x)
      (or (AQ ?w0 ?y) (NPED ?w0 ?y))))))

; Ground Axioms:

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; (A42) (qt(x,y)  $\sqcap$  qt(y,z))  $\sqcap$  qt(x,z)
(forall (?w0 ?x ?y ?z)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?z)
    (qt ?w0 ?x ?y)
    (qt ?w0 ?y ?z))
    (qt ?w0 ?x ?z)))

; (A43) (dqt(x,y)  $\sqcap$  dqt(x,y'))  $\sqcap$  y=y'
(forall (?w0 ?x ?y ?z)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?z)
    (qt ?w0 ?x ?y)
    (qt ?w0 ?x ?z))
    (= ?y ?z)))

; (A44) (qt( $\sqcup$ , x, y)  $\sqcap$  qt( $\sqcup$ , x', y))  $\sqcap$  x=x'
(forall (?w0 ?f ?x ?y ?z)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?z)
    (qtf ?w0 ?f ?x ?y)
    (qtf ?w0 ?f ?z ?y))
    (= ?x ?z)))

; (A45) (qt( $\sqcup$ , x, y)  $\sqcap$  qt( $\sqcup$ , y, z))  $\sqcap$  DJ( $\sqcup$ ,  $\sqcup$ )
(forall (?w0 ?f ?g ?x ?y ?z)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?z)
    (qtf ?w0 ?f ?x ?y)
    (qtf ?w0 ?g ?y ?z))
    (DJ ?w0 ?f ?g)))

; Existential Axioms:
; (A46) TQ(x)  $\sqcap$   $\sqcup$ !y(qt(x,y)  $\sqcap$  PD(y))
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (TQ ?w0 ?x))
    (exists (?y)
      (and (PARTICULAR ?y)
        (qt ?w0 ?x ?y)
        (PD ?w0 ?y)
        (forall (?z)
          (=> (and (PARTICULAR ?z)
            (qt ?w0 ?x ?z)
            (PD ?w0 ?z))
            (= ?z ?y)))))))

; (A47) PQ(x)  $\sqcap$   $\sqcup$ !y(qt(x,y)  $\sqcap$  PED(y))
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (PQ ?w0 ?x))
    (exists (?y)
      (and (PARTICULAR ?y)
        (qt ?w0 ?x ?y)
        (PED ?w0 ?y)
        (forall (?z)
          (=> (and (PARTICULAR ?z)
            (qt ?w0 ?x ?z)
            (PED ?w0 ?z))
            (= ?z ?y)))))))

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(= ?z ?y)))))))))

; (A48)  $AQ(x) \sqsubseteq \exists!y(qt(x, y) \sqsubseteq NPED(y))$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (AQ ?w0 ?x)
    (exists (?y)
      (and (PARTICULAR ?y)
        (qt ?w0 ?x ?y)
        (NPED ?w0 ?y)
        (forall (?z)
          (=> (and (PARTICULAR ?z)
            (qt ?w0 ?x ?z)
            (NPED ?w0 ?z))
          (= ?z ?y))))))))))

; (A49)  $PD(x) \sqsubseteq \exists y(qt(TL, y, x))$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (PD ?w0 ?x)
    (exists (?y)
      (and (PARTICULAR ?y) (qtf ?w0 TL ?y ?x))))))

; (A50)  $PED(x) \sqsubseteq \exists y(qt(SL, y, x))$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (PED ?w0 ?x)
    (exists (?y)
      (and (PARTICULAR ?y) (qtf ?w0 SL ?y ?x))))))

; (A51)  $NPED(x) \sqsubseteq \exists!y(SBL(AQ, \Box) \sqsubseteq qt(\Box, y, x))$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (NPED ?w0 ?x)
    (exists (?f ?y)
      (and (PARTICULAR ?y)
        (UNIVERSAL ?f)
        (SBL ?w0 AQ ?f)
        (qtf ?w0 ?f ?y ?x))))))

; Quale
; Immediate Quale
; Argument restrictions:
; (A52)  $ql(x, y) \sqsubseteq (TR(x) \sqsubseteq TQ(y))$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (ql ?w0 ?x ?y))
    (and (TR ?w0 ?x) (TQ ?w0 ?y))))

; (A53)  $(ql(x, y) \sqsubseteq TL(y)) \sqsubseteq T(x)$ 
(forall (?w0 ?x ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (ql ?w0 ?x ?y)
    (TL ?w0 ?y))
    (T ?w0 ?x)))

; Basic Axioms:
; (A54)  $(ql(x, y) \sqsubseteq ql(x', y)) \sqsubseteq x = x'$ 
(forall (?w0 ?x ?x1 ?y)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?x1)
    (PARTICULAR ?y)
    (ql ?w0 ?x ?y)
    (ql ?w0 ?x1 ?y))
    (= ?x ?x1)))

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; Existential Axioms:
; (A55)  $\neg \text{ql}(y, x) \rightarrow \neg \text{ql}(y, x)$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (TQ ?w0 ?x))
    (exists (?y)
      (and (PARTICULAR ?y) (ql ?w0 ?y ?x))))))

; (A56)  $(L_{\Box}(\Box \neg \Box(x) \neg \Box(y) \neg \text{ql}(r, x) \neg \text{ql}(r', y)) \rightarrow \Box \Box (L_{\Box}(\Box \neg \Box(r) \neg \Box(r'))$ 
(forall (?w0 ?f ?x ?y ?r ?r1)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?r)
    (PARTICULAR ?r1)
    (L.X ?w0 ?f)
    (?f ?w0 ?x)
    (?f ?w0 ?y)
    (ql ?w0 ?r ?x)
    (ql ?w0 ?r1 ?y))
    (exists (?g)
      (and (UNIVERSAL ?g)
        (L.X ?w0 ?g)
        (?g ?w0 ?r)
        (?g ?w0 ?r1))))))

; (A57)  $(L_{\Box}(\Box \neg \Box(x) \neg \Box(y) \neg \text{ql}(r, x) \neg \text{ql}(r', y)) \rightarrow \Box \Box \Box (L_{\Box}(\Box \neg \Box(r) \neg \Box(r'))$ 
(forall (?w0 ?f ?x ?y ?r ?r1)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?r)
    (PARTICULAR ?r1)
    (L.X ?w0 ?f)
    (?f ?w0 ?x)
    (not (?f ?w0 ?y))
    (ql ?w0 ?r ?x)
    (ql ?w0 ?r1 ?y))
    (not (exists (?g)
      (and (UNIVERSAL ?g)
        (L.X ?w0 ?g)
        (?g ?w0 ?r)
        (?g ?w0 ?r1))))))

; Temporary Quale
; Argument restrictions:
; (A58)  $\text{ql}(x, y, t) \rightarrow ((PR(x) \rightarrow AR(x)) \rightarrow (PQ(y) \rightarrow AQ(y)) \rightarrow T(t))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (ql ?w0 ?x ?y ?t))
    (and (or (PR ?w0 ?x) (AR ?w0 ?x))
      (or (PQ ?w0 ?y) (AQ ?w0 ?y))
      (T ?w0 ?t))))))

; (A59)  $\text{ql}(x, y, t) \rightarrow (PR(x) \rightarrow PQ(y))$ 
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (ql ?w0 ?x ?y ?t))
    (<=> (PR ?w0 ?x) (PQ ?w0 ?y))))

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; (A60) ql(x, y, t)  $\square$  (AR(x)  $\square$  AQ(y))
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (ql ?w0 ?x ?y ?t))
    (<=> (AR ?w0 ?x) (AQ ?w0 ?y))))

; (A61) (ql(x, y, t)  $\square$  SL(y))  $\square$  S(x)
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (ql ?w0 ?x ?y ?t)
    (SL ?w0 ?y))
    (S ?w0 ?x)))

; Existential Axioms:
; (A62) ((PQ(x) AQ(x))  $\square$  PRE(x, t))  $\square$   $\square$ y(ql(y, x, t))
(forall (?w0 ?x)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (or (PQ ?w0 ?x) (AQ ?w0 ?x))
    (PRE ?w0 ?x ?t))
    (exists (?y)
      (and (PARTICULAR ?y) (ql ?w0 ?y ?x ?t)))))

; (A63) (L $\square$ ( $\square$ )  $\square$   $\square$ (x)  $\square$   $\square$ (y)  $\square$  ql(r, x, t)  $\square$  ql(r', y, t))  $\square$   $\square$  $\square$ (L $\square$ ( $\square$ )  $\square$   $\square$ (r)  $\square$   $\square$ (r'))
(forall (?w0 ?f ?x ?y ?r ?r1 ?t)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?r)
    (PARTICULAR ?r1)
    (PARTICULAR ?t)
    (L.X ?w0 ?f)
    (?f ?w0 ?x)
    (?f ?w0 ?y)
    (ql ?w0 ?r ?x ?t)
    (ql ?w0 ?r1 ?y ?t))
    (exists (?g)
      (and (UNIVERSAL ?g)
        (L.X ?w0 ?g)
        (?g ?w0 ?r)
        (?g ?w0 ?r1)))))

; (A64) (L $\square$ ( $\square$ )  $\square$   $\square$ (x)  $\square$   $\square$ (y)  $\square$  ql(r, x, t)  $\square$  ql(r', y, t))  $\square$   $\square$  $\square$  $\square$ (L $\square$ ( $\square$ )  $\square$   $\square$ (r)  $\square$   $\square$ (r'))
(forall (?w0 ?f ?x ?y ?r ?r1 ?t)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?r)
    (PARTICULAR ?r1)
    (PARTICULAR ?t)
    (L.X ?w0 ?f)
    (?f ?w0 ?x)
    (not (?f ?w0 ?y))
    (ql ?w0 ?r ?x ?t)
    (ql ?w0 ?r1 ?y ?t))
    (not (exists (?g)
      (and (UNIVERSAL ?g)
        (L.X ?w0 ?g)
        (?g ?w0 ?r)

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(?g ?w0 ?r1))))))

; Link with Parthood and extension:
; (A65) ql(x, y, t)  $\sqsubseteq$  PRE(y, t)
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (ql ?w0 ?x ?y ?t))
    (PRE ?w0 ?y ?t)))

; (A66) ql(x, y, t)  $\sqsubseteq$   $\sqsubseteq$  t'(P(t', t)  $\sqsubseteq$  ql(x, y, t'))
(forall (?w0 ?x ?y ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t))
    (<=> (ql ?w0 ?x ?y ?t)
      (forall (?u)
        (=> (and (PARTICULAR ?u) (P ?w0 ?u ?t))
          (ql ?w0 ?x ?y ?u)))))))

; Dependence and Spatial Dependence
; Links between categories
; (A67) MSD(TQ, PD)
(forall (?w0) (=> (WORLD ?w0) (MSD ?w0 TQ PD)))

; (A68) MSDs(PQ, PED)
(forall (?w0) (=> (WORLD ?w0) (MSD.S ?w0 PQ PED)))

; (A69) MSD(AQ, NPED)
(forall (?w0) (=> (WORLD ?w0) (MSD ?w0 AQ NPED)))

; (A70) OGD(F, NAPO)
(forall (?w0) (=> (WORLD ?w0) (OGD ?w0 F NAPO)))

; (A71) OSD(MOB, APO)
(forall (?w0) (=> (WORLD ?w0) (OSD ?w0 MOB APO)))

; (A72) OGD(SAG, APO)
(forall (?w0) (=> (WORLD ?w0) (OGD ?w0 SAG APO)))

; (A73) OGD(NASO, SC)
(forall (?w0) (=> (WORLD ?w0) (OGD ?w0 NASO SC)))

; (A74) OD(NPED, PED)
(forall (?w0) (=> (WORLD ?w0) (OD ?w0 NPED PED)))

; Characterization of Categories
; Perdurant
; Conditions on Perdurant's Leaves
; (A75) PSBL(ACH,  $\square$ )  $\sqsubseteq$  (NEPs( $\square$ )  $\sqsubseteq$  CM-( $\square$ )  $\sqsubseteq$  AT( $\square$ ))
(forall (?w0 ?f)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PSBL ?w0 ACH ?f))
    (and (NEP.S ?w0 ?f) (CM~ ?w0 ?f) (AT ?w0 ?f))))

; (A76) PSBL(ACC,  $\square$ )  $\sqsubseteq$  (NEPs( $\square$ )  $\sqsubseteq$  CM-( $\square$ )  $\sqsubseteq$  AT-( $\square$ ))
(forall (?w0 ?f)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (PSBL ?w0 ACC ?f))
    (and (NEP.S ?w0 ?f) (CM~ ?w0 ?f) (AT~ ?w0 ?f))))

; (A77) PSBL(ST,  $\square$ )  $\sqsubseteq$  (NEPs( $\square$ )  $\sqsubseteq$  CM( $\square$ )  $\sqsubseteq$  HOM( $\square$ ))
(forall (?w0 ?f)

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(=> (and (WORLD ?w0)
          (UNIVERSAL ?f)
          (PSBL ?w0 ST ?f))
      (and (NEP.S ?w0 ?f) (CM ?w0 ?f) (HOM ?w0 ?f))))

; (A78) PSBL(PRO,  $\Box$ )  $\Box$  (NEPs( $\Box$ )  $\Box$  CM( $\Box$ )  $\Box$  HOM~( $\Box$ ))
(forall (?w0 ?f)
  (=> (and (WORLD ?w0)
            (UNIVERSAL ?f)
            (PSBL ?w0 PRO ?f))
      (and (NEP.S ?w0 ?f) (CM ?w0 ?f) (HOM~ ?w0 ?f)))))

; Existential Axioms
; (A79)  $\Box\Box$ (PSBL(ACH,  $\Box$ ))
(forall (?w0)
  (=> (WORLD ?w0)
      (exists (?f) (and (UNIVERSAL ?f) (PSBL ?w0 ACH ?f)))))

; (A80)  $\Box\Box$ (PSBL(ACC,  $\Box$ ))
(forall (?w0)
  (=> (WORLD ?w0)
      (exists (?f) (and (UNIVERSAL ?f) (PSBL ?w0 ACC ?f)))))

; (A81)  $\Box\Box$ (PSBL(ST,  $\Box$ ))
(forall (?w0)
  (=> (WORLD ?w0)
      (exists (?f) (and (UNIVERSAL ?f) (PSBL ?w0 ST ?f)))))

; (A82)  $\Box\Box$ (PSBL(PRO,  $\Box$ ))
(forall (?w0)
  (=> (WORLD ?w0)
      (exists (?f) (and (UNIVERSAL ?f) (PSBL ?w0 PRO ?f)))))
; =====
; THEOREMS
; General Properties
; (T1)  $\Box$ K(x, x, t)
(forall (?w0 ?x ?t)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (PARTICULAR ?t))
      (not (K ?w0 ?x ?x ?t)))))

; (T2) SK( $\Box$ ,  $\Box$ )  $\Box$  SD( $\Box$ ,  $\Box$ )
(forall (?w0 ?f ?g)
  (=> (and (WORLD ?w0) (UNIVERSAL ?f) (UNIVERSAL ?g) (SK ?w0 ?f ?g))
      (SD ?w0 ?f ?g))))

; (T3) GK( $\Box$ ,  $\Box$ )  $\Box$  GD( $\Box$ ,  $\Box$ )
(forall (?w0 ?f ?g)
  (=> (and (WORLD ?w0) (UNIVERSAL ?f) (UNIVERSAL ?g) (GK ?w0 ?f ?g))
      (GD ?w0 ?f ?g))))

; (T4) (SK( $\Box$ ,  $\Box$ )  $\Box$  SK( $\Box$ ,  $\Box$ )  $\Box$  DJ( $\Box$ ,  $\Box$ ))  $\Box$  SK( $\Box$ ,  $\Box$ )
(forall (?w0 ?f ?g ?h)
  (=> (and (WORLD ?w0)
            (UNIVERSAL ?f)
            (UNIVERSAL ?g)
            (UNIVERSAL ?h)
            (SK ?w0 ?f ?g)
            (SK ?w0 ?g ?h)
            (DJ ?w0 ?f ?h))
      (SK ?w0 ?f ?h))))

; (T5) (GK( $\Box$ ,  $\Box$ )  $\Box$  GK( $\Box$ ,  $\Box$ )  $\Box$  DJ( $\Box$ ,  $\Box$ ))  $\Box$  GK( $\Box$ ,  $\Box$ )
(forall (?w0 ?f ?g ?h)
  (=> (and (WORLD ?w0)
            (UNIVERSAL ?f)
            (UNIVERSAL ?g)
            (UNIVERSAL ?h)
            (GK ?w0 ?f ?g)
            (GK ?w0 ?g ?h)
            (DJ ?w0 ?f ?h))
      (GK ?w0 ?f ?h))))

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        (UNIVERSAL ?h)
        (GK ?w0 ?f ?g)
        (GK ?w0 ?g ?h)
        (DJ ?w0 ?f ?h))
    (GK ?w0 ?f ?h)))

; Ground Properties
; (T6)  $\Box PC(x, x, t)$ 
(forall (?w0 ?x ?t)
  (=> (and (WORLD ?w0) (PARTICULAR ?x) (PARTICULAR ?t))
    (not (PC ?w0 ?x ?x ?t)))))

; (T7)  $PC(x, y, t) \Box \Box PC(y, x, t)$ 
(forall (?w0 ?x ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?y)
    (PARTICULAR ?t)
    (PC ?w0 ?x ?y ?t))
    (not (PC ?w0 ?y ?x ?t)))))

; (T8)  $\Box qt(x, x)$ 
(forall (?w0 ?x)
  (=> (and (WORLD ?w0) (PARTICULAR ?x))
    (not (qt ?w0 ?x ?x)))))

; General properties
; (T9)  $(SD(\Box, \Box) \Box SD(\Box, \Box) \Box DJ(\Box, \Box)) \Box SD(\Box, \Box)$ 
(forall (?w0 ?f ?g ?h)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (UNIVERSAL ?h)
    (SD ?w0 ?f ?g)
    (SD ?w0 ?g ?h)
    (DJ ?w0 ?f ?h))
    (SD ?w0 ?f ?h)))

; (T10)  $(GD(\Box, \Box) \Box GD(\Box, \Box) \Box DJ(\Box, \Box)) \Box GD(\Box, \Box)$ 
(forall (?w0 ?f ?g ?h)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (UNIVERSAL ?h)
    (GD ?w0 ?f ?g)
    (GD ?w0 ?g ?h)
    (DJ ?w0 ?f ?h))
    (GD ?w0 ?f ?h)))

; (T11)  $(SD(\Box, \Box) \Box GD(\Box, \Box) \Box DJ(\Box, \Box)) \Box GD(\Box, \Box)$ 
(forall (?w0 ?f ?g ?h)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (UNIVERSAL ?h)
    (SD ?w0 ?f ?g)
    (GD ?w0 ?g ?h)
    (DJ ?w0 ?f ?h))
    (GD ?w0 ?f ?h)))

; (T12)  $(GD(\Box, \Box) \Box SD(\Box, \Box) \Box DJ(\Box, \Box)) \Box GD(\Box, \Box)$ 
(forall (?w0 ?f ?g ?h)
  (=> (and (WORLD ?w0)
    (UNIVERSAL ?f)
    (UNIVERSAL ?g)
    (UNIVERSAL ?h)
    (GD ?w0 ?f ?g)
    (SD ?w0 ?g ?h)
    (DJ ?w0 ?f ?h))
    (GD ?w0 ?f ?g)))

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        (SD ?w0 ?g ?h)
        (DJ ?w0 ?f ?h))
    (GD ?w0 ?f ?h)))

; (T13) SDS(□ □) □ SD(□ □)
(forall (?w0 ?f ?g)
  (=> (and (WORLD ?w0) (UNIVERSAL ?f) (UNIVERSAL ?g) (SD.S ?w0 ?f ?g))
    (SD ?w0 ?f ?g))))

; (T14) GDS(□ □) □ GD(□ □)
(forall (?w0 ?f ?g)
  (=> (and (WORLD ?w0) (UNIVERSAL ?f) (UNIVERSAL ?g) (GD.S ?w0 ?f ?g))
    (GD ?w0 ?f ?g))))

; Being Present
; (T15) (ED(x) PD(x) Q(x)) □ □(PRE(x, t))
(forall (?w0 ?x)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (or (ED ?w0 ?x) (PD ?w0 ?x) (Q ?w0 ?x)))
    (exists (?t)
      (and (PARTICULAR ?t) (PRE ?w0 ?x ?t))))))

; (T16) ((PED(x) PQ(x)) □ PRE(x, t)) □ □s(PRE(s, x, t))
(forall (?w0 ?x ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?t)
    (or (PED ?w0 ?x) (PQ ?w0 ?x))
    (PRE ?w0 ?x ?t))
    (exists (?s)
      (and (PARTICULAR ?s) (PRE ?w0 ?s ?x ?t))))))

; (T17) (PRE(x, t) □ P(t', t)) □ PRE(x, t')
(forall (?w0 ?x ?t ?t1)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?t)
    (PARTICULAR ?t1)
    (PRE ?w0 ?x ?t)
    (P ?w0 ?t1 ?t))
    (PRE ?w0 ?x ?t1))))

; (T18) PRE(s, x, t) □ PRE(x, t)
(forall (?w0 ?x ?s ?t)
  (=> (and (WORLD ?w0)
    (PARTICULAR ?x)
    (PARTICULAR ?s)
    (PARTICULAR ?t)
    (PRE ?w0 ?s ?x ?t))
    (PRE ?w0 ?x ?t))))

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